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Exam Maths 2 - lasts 1:30 minutes   Mai 12, 2024

Name:...............
Group:..............

Exercise 1 ***(9 pts) Part I*** *Choose the right answer. Let*

$$I = \int_1^e \frac{1}{x}\ln x dx \ , \quad J = \int \frac{1}{x^2+x}dx \ , \ K = \int x\ln x dx \ ,$$
$$L = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x}dx \quad \text{and} \quad M = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x+\cos x}dx$$

1- *The value of I is*

. $I = -\frac{1}{2}$ ☒ $I = \frac{1}{2}$. $I = e$. $I = 1$

2- *The integral J is*

. $J = \ln\left|x^2+x\right| + C$. $J = \ln(x+1) - \ln x + C$ ☒ $J = \ln\left|\frac{x}{x+1}\right| + C$

3- *The integral K is*

☒ $\mathbf{K} = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$. $\mathbf{K} = \frac{1}{2}x^2\ln x + \frac{1}{4}x^2 + C$. $K = \frac{1}{2}x^2\ln x + C$

4. *The value of $L+M$ and $L-M$ are*

☒ $\begin{cases} L+M = \frac{\pi}{2} \\ L-M = 0 \end{cases}$. $\begin{cases} L+M = 0 \\ L-M = \frac{\pi}{2} \end{cases}$. $\begin{cases} L+M = \frac{\pi}{2} \\ L-M = \frac{\pi}{2} \end{cases}$. $\begin{cases} L+M = \frac{\pi}{2} \\ L-M = -\frac{\pi}{2} \end{cases}$

5. *The value of K and L are*

☒ $\begin{cases} L = \frac{\pi}{4} \\ M = \frac{\pi}{4} \end{cases}$. $\begin{cases} L = -\frac{\pi}{4} \\ M = \frac{\pi}{4} \end{cases}$. $\begin{cases} L = 0 \\ M = \frac{\pi}{2} \end{cases}$. $\begin{cases} L = \frac{\pi}{2} \\ M = -\frac{\pi}{2} \end{cases}$

Part II *Let the following diferentiabl equations*

$$y' - xy = -x \qquad (1)$$
$$2y'' - 3y' + y = 0 \qquad (2)$$

6. *The homogenous somutions of 1 are*

. $y_H = Ce^{-x}$. $y_H = Ce^{x}$. $y_H = Ce^{-\frac{1}{2}x^x}$ ☒ $y_H = Ce^{\frac{1}{2}x^x}$

7- The partucular somution of 1 is

☒ $y_p = 1$. $y_p = x$. $y_p = xe^x$. $y_p = -x$

8- The homogenous solutions of 2 are

☒ $y_H = C_1 e^x + C_2 e^{\frac{1}{2}x}$. $y_H = C_1 e^x + C_2 e^{-x}$. $y_H = C_1 e^x + C_2 e^{-\frac{1}{2}x}$

9. *The limited developement of* $f(x) = (e^x - x)(x + \sin x)$ *in order 3 is*

☒ $f(x) = 2x + \frac{5}{6}x^3 + x^3 \varepsilon(x)$. $f(x) = 2x - \frac{5}{6}x^3 + x^3 \varepsilon(x)$. $f(x) = \frac{5}{6}x^3 + x^3 \varepsilon(x)$

Exercise 2 ***6 pts*** *Consider the folloiwng matrix*

$$A = \begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix} \text{ and } P = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}.$$

1. *The linear map* $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ *with respect to the standard base* $\mathbb{R}^2$ *is*

. $f(x, y) = (3x - 2y, x + y)$ ☒ $f(x, y) = (3x - 2y, x)$. $f(x, y) = (3x - 2y, y)$

2- The inverse matrix of P is

☒ $P^{-1} = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}$. $P^{-1} = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}$. $P^{-1} = \begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix}$

3- The result of $A^2 - 3A$ *is*

. $A^2 - 3A = 2I_2$. $A^2 - 3A = I_2$ ☒ $A^2 - 3A = -2I_2$

4. *The eigenvalues of A are*

☒ $\lambda_1 = 1$ *or* $\lambda_2 = 2$. $\lambda_1 = 1$ *or* $\lambda_2 = -2$. $\lambda_1 = -1$ *or* $\lambda_2 = 2$

5. *The result of* $P^{-1}AP$ *is*

. $P^{-1}AP = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ☒ $P^{-1}AP = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$. $P^{-1}AP = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

6. *The matrix* A^n, *for all* $n \geq 1$.*is*

☒ $A^n = \begin{pmatrix} 2 \times 2^n - 1 & 2 - 2 \times 2^n \\ 2^n - 1 & 2 - 2^n \end{pmatrix}$. $A^n = \begin{pmatrix} 2 \times 2^n + 1 & 2 - 2 \times 2^n \\ 2^n - 1 & 2 - 2^n \end{pmatrix}$. $A^n = \begin{pmatrix} 3^n & -2^n \\ 1 & 0 \end{pmatrix}$

Exercise 3 ***5 pts*** *Show that the solutions of the following differential equation*

$$y' + 2y = (x + 1)\sqrt{y} \qquad (3)$$

is

$$y = \left(\frac{x}{2} + Ce^{-x}\right)^2.$$

Solution 4 *This is a Bernoulli differentiable equation 3 , where* $\alpha = \frac{1}{2}$. *We first divided the equation throught by* $\sqrt{y}$, *thereby expressing it in the equivalent form*

$$\frac{y'}{\sqrt{y}} + 2\frac{y}{\sqrt{y}} = (x+1) \ldots\ldots \textbf{\textit{1pt}} \tag{4}$$

by using the change variable $z = y^{1-\frac{1}{2}}$, *then* $z' = \frac{1}{2}\frac{y'}{\sqrt{y}}$ *and equation 4 transforms into*

$$2z' + 2z = x + 1 \ldots\ldots \textbf{\textit{1pt}} \tag{5}$$

the solution of linear differential equation of 1st order 5 is

$$z = \frac{x}{2} + Ce^{-x} \ldots\ldots \textbf{\textit{1pt}}$$

Thus we obtain the solutions of **??** *in then form*

$$y = \left(\frac{x}{2} + Ce^{-x}\right)^2 \ldots\ldots \textbf{\textit{2pt}}$$