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Exam Maths 2 — lasts 1:30 minutes — Mai 12, 2024

Name:.....

Group:.....

Exercise 1 (9 pts) Part I Choose the right answer. Let

$$I = \int_1^e \frac{1}{x} \ln x dx, \quad J = \int \frac{1}{x^2 + x} dx, \quad K = \int x \ln x dx,$$

$$L = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx \quad \text{and} \quad M = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x} dx$$

1- The value of I is

$I = -\frac{1}{2}$ $I = \frac{1}{2}$ $I = e$ $I = 1$

2- The integral J is

$J = \ln|x^2 + x| + C$ $J = \ln(x+1) - \ln x + C$ $J = \ln\left|\frac{x}{x+1}\right| + C$

3- The integral K is

$K = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$ $K = \frac{1}{2}x^2 \ln x + \frac{1}{4}x^2 + C$ $K = \frac{1}{2}x^2 \ln x + C$

4. The value of $L + M$ and $L - M$ are

$\begin{cases} L + M = \frac{\pi}{2} \\ L - M = 0 \end{cases}$ $\begin{cases} L + M = 0 \\ L - M = \frac{\pi}{2} \end{cases}$ $\begin{cases} L + M = \frac{\pi}{2} \\ L - M = \frac{\pi}{2} \end{cases}$ $\begin{cases} L + M = \frac{\pi}{2} \\ L - M = -\frac{\pi}{2} \end{cases}$

5. The value of K and L are

$\begin{cases} L = \frac{\pi}{4} \\ M = \frac{\pi}{4} \end{cases}$ $\begin{cases} L = -\frac{\pi}{4} \\ M = \frac{\pi}{4} \end{cases}$ $\begin{cases} L = 0 \\ M = \frac{\pi}{2} \end{cases}$ $\begin{cases} L = \frac{\pi}{2} \\ M = -\frac{\pi}{2} \end{cases}$

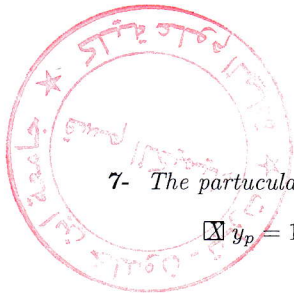
Part II Let the following differentiable equations

$$y' - xy = -x \quad (1)$$

$$2y'' - 3y' + y = 0 \quad (2)$$

6. The homogenous solutions of 1 are

$y_H = Ce^{-x}$ $y_H = Ce^x$ $y_H = Ce^{-\frac{1}{2}x^2}$ $y_H = Ce^{\frac{1}{2}x^2}$



7- The particular solution of 1 is

$y_p = 1$ $y_p = x$ $y_p = xe^x$ $y_p = -x$

8- The homogenous solutions of 2 are

$y_H = C_1 e^x + C_2 e^{\frac{1}{2}x}$ $y_H = C_1 e^x + C_2 e^{-x}$ $y_H = C_1 e^x + C_2 e^{-\frac{1}{2}x}$

9. The limited development of $f(x) = (e^x - x)(x + \sin x)$ in order 3 is

$f(x) = 2x + \frac{5}{6}x^3 + x^3 \varepsilon(x)$ $f(x) = 2x - \frac{5}{6}x^3 + x^3 \varepsilon(x)$ $f(x) = \frac{5}{6}x^3 + x^3 \varepsilon(x)$

Exercise 2 6 pts Consider the following matrix

$$A = \begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix} \text{ and } P = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}.$$

1. The linear map $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ with respect to the standard base \mathbb{R}^2 is

$f(x, y) = (3x - 2y, x + y)$ $f(x, y) = (3x - 2y, x)$ $f(x, y) = (3x - 2y, y)$

2- The inverse matrix of P is

$P^{-1} = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}$ $P^{-1} = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}$ $P^{-1} = \begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix}$

3- The result of $A^2 - 3A$ is

$A^2 - 3A = 2I_2$ $A^2 - 3A = I_2$ $A^2 - 3A = -2I_2$

4. The eigenvalues of A are

$\lambda_1 = 1$ or $\lambda_2 = 2$ $\lambda_1 = 1$ or $\lambda_2 = -2$ $\lambda_1 = -1$ or $\lambda_2 = 2$

5. The result of $P^{-1}AP$ is

$P^{-1}AP = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $P^{-1}AP = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ $P^{-1}AP = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

6. The matrix A^n , for all $n \geq 1$ is

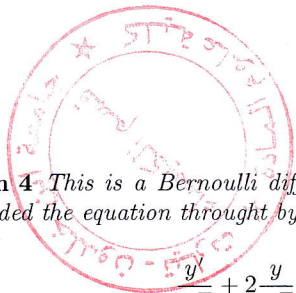
$A^n = \begin{pmatrix} 2 \times 2^n - 1 & 2 - 2 \times 2^n \\ 2^n - 1 & 2 - 2^n \end{pmatrix}$ $A^n = \begin{pmatrix} 2 \times 2^n + 1 & 2 - 2 \times 2^n \\ 2^n - 1 & 2 - 2^n \end{pmatrix}$ $A^n = \begin{pmatrix} 3^n & -2^n \\ 1 & 0 \end{pmatrix}$

Exercise 3 5 pts Show that the solutions of the following differential equation

$$y' + 2y = (x + 1)\sqrt{y} \quad (3)$$

is

$$y = \left(\frac{x}{2} + Ce^{-x} \right)^2.$$



Solution 4 This is a Bernoulli differentiable equation 3 , where $\alpha = \frac{1}{2}$. We first divided the equation through by \sqrt{y} , thereby expressing it in the equivalent form

$$\frac{y'}{\sqrt{y}} + 2\frac{y}{\sqrt{y}} = (x+1) \dots 1pt \quad (4)$$

by using the change variable $z = y^{1-\frac{1}{2}}$, then $z' = \frac{1}{2}\frac{y'}{\sqrt{y}}$ and equation 4 transforms into

$$2z' + 2z = x+1 \dots 1pt \quad (5)$$

the solution of linear differential equation of 1st order 5 is

$$z = \frac{x}{2} + Ce^{-x} \dots 1pt$$

Thus we obtain the solutions of ?? in then form

$$y = \left(\frac{x}{2} + Ce^{-x}\right)^2 \dots 2pt$$